**Exercises for Section 1.1**

2. If you wanted to estimate the mean height of all the students at a university, which one of the following sampling strategies would be best? Why? Note that none of the methods are true simple random samples.
   i. Measure the heights of 50 students found in the gym during basketball intramurals.
   ii. Measure the heights of all engineering majors.
   iii. Measure the heights of the students selected by choosing the first name on each page of the campus phone book.

3. True or false:
   a. A simple random sample is guaranteed to reflect exactly the population from which it was drawn.
   b. A simple random sample is free from any systematic tendency to differ from the population from which it was drawn.

4. A quality engineer draws a simple random sample of 50 O-rings from a lot of several thousand. She measures the thickness of each, and finds that 45 of them, or 90%, meet a certain specification. Which of the following statements is correct?
   i. The proportion of O-rings in the entire lot that meet the specification is likely to be equal to 90%.
   ii. The proportion of O-rings in the entire lot that meet the specification is likely to be close to 90% but is not likely to equal 90%.

**Exercises for Section 1.2**

4. Is the sample median always equal to one of the values in the sample? If so, explain why. If not, give an example.

8. In a certain company, every worker received a $50-per-week raise. How does this affect the mean salary? The standard deviation of the salaries?

11. In a statistics class of 30 students, the mean score on the midterm was 72. In another class of 40 students, the mean score was 79. What was the mean for the two classes combined?

14. A list of 10 numbers has a mean of 20, a median of 18, and a standard deviation of 5. The largest number on the list is 39.27. By accident, this number is changed to 392.7.
   a. What is the value of the mean after the change?
   b. What is the value of the median after the change?
   c. What is the value of the standard deviation after the change?

6. Sketch a histogram for which
   a. The mean is greater than the median.
   b. The mean is less than the median.
   c. The mean is approximately equal to the median.

**Exercises for Section 1.3**

2. Following is a list of the number of hazardous waste sites in each of the 50 states of the United States as of April 1995. The data are taken from *The World Almanac and Book of Facts 1996* (World Almanac Books, Mahwah, NJ, 1996). The list has been sorted into numerical order.

```
1 2 3 4 4 5 6 8 8 9
10 10 10 11 11 11 12 12 12 12
13 13 14 15 16 17 17 18 18 19
19 20 22 23 24 25 29 30 33 37
38 39 40 55 58 77 81 96 102 107
```

a. Construct a stem-and-leaf plot for these data.
b. Construct a histogram for these data.
c. Construct a dotplot for these data.
d. Construct a boxplot for these data. Does the boxplot show any outliers?

8. The following histogram presents the amounts of silver [in parts per million (ppm)] found in a sample of rocks. One rectangle from the histogram is missing. What is its height?
11. An engineer wants to draw a boxplot for the following sample:
   37 82 20 25 31 10 41 44 4 36 68
Which of these values, if any, will be labeled as outliers?

12. Which of the following statistics cannot be determined from a boxplot?
   i. The median
   ii. The mean
   iii. The first quartile
   iv. The third quartile
   v. The interquartile range

14. Following are boxplots comparing the charge [in coulombs per mole (C/mol) × 10⁻²⁵] at pH 4.0 and pH 4.5 for a collection of proteins (from the article “Optimal Synthesis of Protein Purification Processes,” E. Vasquez-Alvarex, M. Linquiao, and J. Pinto, Biotechnology Progress 2001:685–695). True or false:
   a. The median charge for the pH of 4.0 is greater than the 75th percentile of charge for the pH of 4.5.
   b. Approximately 25% of the charges for pH 4.5 are less than the smallest charge at pH 4.0.

16. Match each histogram to the boxplot that represents the same data set.
18. Match each scatterplot to the statement that best describes it.

i. The relationship between $x$ and $y$ is approximately linear.
ii. The relationship between $x$ and $y$ is nonlinear.
iii. There isn't much of any relationship between $x$ and $y$.
iv. The relationship between $x$ and $y$ is approximately linear, except for an outlier.
Exercises for Section 2.1

2. An octahedral die (eight faces) has the number 1 painted on two of its faces, the number 2 painted on three of its faces, the number 3 painted on two of its faces, and the number 4 painted on one face. The die is rolled. Assume that each face is equally likely to come up.

4. A commuter passes through three traffic lights on the way to work. Each light is either red, yellow, or green. An experiment consists of observing the colors of the three lights.
   a. List the 27 outcomes in the sample space.
   b. Let $A$ be the event that all the colors are the same. List the outcomes in $A$.
   c. Let $B$ be the event that all the colors are different. List the outcomes in $B$.
   d. Let $C$ be the event that at least two lights are green. List the outcomes in $C$.
   e. List the outcomes in $A \cap C$.
   f. List the outcomes in $A \cup B$.
   g. List the outcomes in $A \cap C'$.
   h. List the outcomes in $A' \cap C$.
   i. Are events $A$ and $C$ mutually exclusive? Explain.
   j. Are events $B$ and $C$ mutually exclusive? Explain.

8. An item manufactured by a certain process has probability 0.10 of being defective. True or false:
   a. If a sample of 100 items is drawn, exactly 10 of them will be defective.
   b. If a sample of 100 items is drawn, the number of defectives is likely to be close to 10, but not exactly equal to 10.
   c. As more and more items are sampled, the proportion of defective items will approach 10%.

10. Let $E$ be the event that a new car requires engine work under warranty and let $T$ be the event that the car requires transmission work under warranty. Suppose that $P(E) = 0.10$, $P(T) = 0.02$, and $P(E \cap T) = 0.01$.
   a. Find the probability that the car needs work on either the engine, the transmission, or both.
   b. Find the probability that neither the engine nor the transmission needs work.
   c. Find the probability that the car needs work on the engine but not on the transmission.

Exercises for Section 2.2

2. A chemical engineer is designing an experiment to determine the effect of temperature, stirring rate, and type of catalyst on the yield of a certain reaction. She wants to study five different reaction temperatures, two different stirring rates, and four different catalysts. If each run of the experiment involves a choice of one temperature, one stirring rate, and one catalyst, how many different runs are possible?

4. A group of 10 people have gotten together to play basketball. They will begin by dividing themselves into two teams of 5 players each. In how many ways can this be done?

6. A committee of eight people must choose a president, a vice-president, and a secretary. In how many ways can this be done?

8. In a certain state, license plates consist of three letters followed by three numbers.
   a. How many different license plates can be made?
   b. How many different license plates can be made in which no letter or number appears more than once?
   c. A license plate is chosen at random. What is the probability that no letter or number appears more than once?

10. A company has hired 15 new employees, and must assign 6 to the day shift, 5 to the graveyard shift, and 4 to the night shift. In how many ways can the assignment be made?

12. A drawer contains 6 red socks, 4 green socks, and 2 black socks. Two socks are chosen at random. What is the probability that they match?
Exercises for Section 2.3

1. A box contains 10 fuses. Eight of them are rated at 10 amperes (A) and the other two are rated at 15 A. Two fuses are selected at random.
   a. What is the probability that the first fuse is rated at 15 A?
   b. What is the probability that the second fuse is rated at 15 A, given that the first fuse is rated at 10 A?
   c. What is the probability that the second fuse is rated at 15 A, given that the first fuse is rated at 15 A?

2. Refer to Exercise 1. Fuses are randomly selected from the box, one by one, until a 15 A fuse is selected.
   a. What is the probability that the first two fuses are both 10 A?
   b. What is the probability that a total of two fuses are selected from the box?
   c. What is the probability that more than three fuses are selected from the box?

6. Of all failures of a certain type of computer hard drive, it is determined that in 20% of them only the sector containing the file allocation table is damaged, in 70% of them only nonessential sectors are damaged, and in 10% of the cases both the allocation sector and one or more nonessential sectors are damaged. A failed drive is selected at random and examined.
   a. What is the probability that the allocation sector is damaged?
   b. What is the probability that a nonessential sector is damaged?
   c. If the drive is found to have a damaged allocation sector, what is the probability that some nonessential sectors are damaged as well?
   d. If the drive is found to have a damaged nonessential sector, what is the probability that the allocation sector is damaged as well?
   e. If the drive is found to have a damaged allocation sector, what is the probability that no nonessential sectors are damaged?
   f. If the drive is found to have a damaged nonessential sector, what is the probability that the allocation sector is not damaged?

10. Laura and Philip each fire one shot at a target. Laura has probability 0.5 of hitting the target, and Philip has probability 0.3. The shots are independent.
   a. Find the probability that the target is hit.
   b. Find the probability that the target is hit by exactly one shot.
   c. Given that the target was hit by exactly one shot, find the probability that Laura hit the target.

20. At a certain car dealership, the probability that a customer purchases an SUV is 0.20, and the probability that a customer purchases a black SUV is 0.05. Given that a customer purchases an SUV, what is the probability that it is black?

22. A certain delivery service offers both express and standard delivery. Seventy-five percent of parcels are sent by standard delivery, and 25% are sent by express. Of those sent standard, 80% arrive the next day, and of those sent express, 95% arrive the next day. A record of a parcel delivery is chosen at random from the company’s files.
   a. What is the probability that the parcel was shipped express and arrived the next day?
   b. What is the probability that it arrived the next day?
   c. Given that the package arrived the next day, what is the probability that it was sent express?

26. Refer to Example 2.26. Assume that the proportion of people in the community who have the disease is 0.05.
   a. Given that the test is positive, what is the probability that the person has the disease?
   b. Given that the test is negative, what is the probability that the person does not have the disease?

32. A system contains two components, A and B, connected in series, as shown in the diagram.

Assume A and B function independently. For the system to function, both components must function.
   a. If the probability that A fails is 0.05, and the probability that B fails is 0.03, find the probability that the system functions.
   b. If both A and B have probability \( p \) of failing, what must the value of \( p \) be so that the probability that the system functions is 0.90?
   c. If three components are connected in series, and each has probability \( p \) of failing, what must the value of \( p \) be so that the probability that the system functions is 0.90?
2. The following table presents the probability mass function of the number of defects $X$ in a randomly chosen printed-circuit board.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$p(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

a. Find $P(X < 2)$.
b. Find $P(X \geq 1)$.
c. Find $\mu_X$.
d. Find $\sigma^2_X$.

8. The output of a chemical process is continually monitored to ensure that the concentration remains within acceptable limits. Whenever the concentration drifts outside the limits, the process is shut down and recalibrated. Let $X$ be the number of times in a given week that the process is recalibrated. The following table presents values of the cumulative distribution function $F(x)$ of $X$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$F(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.17</td>
</tr>
<tr>
<td>1</td>
<td>0.53</td>
</tr>
<tr>
<td>2</td>
<td>0.84</td>
</tr>
<tr>
<td>3</td>
<td>0.97</td>
</tr>
<tr>
<td>4</td>
<td>1.00</td>
</tr>
</tbody>
</table>

a. What is the probability that the process is recalibrated fewer than two times during a week?
b. What is the probability that the process is recalibrated more than three times during a week?
c. What is the probability that the process is recalibrated exactly once during a week?
d. What is the probability that the process is not recalibrated at all during a week?
e. What is the most probable number of recalibrations to occur during a week?

10. Microprocessing chips are randomly sampled one by one from a large population, and tested to determine if they are acceptable for a certain application. Ninety percent of the chips in the population are acceptable.

a. What is the probability that the first chip chosen is acceptable?
b. What is the probability that the first chip is unacceptable, and the second is acceptable?
c. Let $X$ represent the number of chips that are tested up to and including the first acceptable chip. Find $P(X = 3)$.
d. Find the probability mass function of $X$.

12. Three components are randomly sampled, one at a time, from a large lot. As each component is selected, it is tested. If it passes the test, a success ($S$) occurs; if it fails the test, a failure ($F$) occurs. Assume that 80% of the components in the lot will succeed in passing the test. Let $X$ represent the number of successes among the three sampled components.

a. What are the possible values for $X$?
b. Find $P(X = 3)$.
c. The event that the first component fails and the next two succeed is denoted by $FSS$. Find $P(FSS)$.
d. Find $P(SFS)$ and $P(SSF)$.
e. Use the results of parts (c) and (d) to find $P(X = 2)$.
f. Find $P(X = 1)$.
g. Find $P(X = 0)$.
h. Find $\mu_X$.
i. Find $\sigma^2_X$.
j. Let $Y$ represent the number of successes if four components are sampled. Find $P(Y = 3)$.

14. Specifications call for the thickness of aluminum sheets that are to be made into cans to be between 8 and 11 thousandths of an inch. Let $X$ be the thickness of an aluminum sheet. Assume the probability density function of $X$ is given by

$$f(x) = \begin{cases} \frac{x}{54} & 6 < x < 12 \\ 0 & \text{otherwise} \end{cases}$$

a. What proportion of sheets will meet the specification?
b. Find the mean thickness of a sheet.
c. Find the variance of the thickness of a sheet.
d. Find the standard deviation of the thickness of a sheet.
e. Find the cumulative distribution function of the thickness.
f. Find the median thickness.
g. Find the 10th percentile of the thickness.
h. A particular sheet is 10 thousandths of an inch thick. What proportion of sheets are thicker than this?

18. The lifetime, in years, of a certain type of fuel cell is a random variable with probability density function

$$f(x) = \begin{cases} \frac{81}{(x + 3)^4} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

a. What is the probability that a fuel cell lasts more than 3 years?
b. What is the probability that a fuel cell lasts between 1 and 3 years?
c. Find the mean lifetime.
d. Find the variance of the lifetimes.
e. Find the cumulative distribution function of the lifetime.
f. Find the median lifetime.
g. Find the 30th percentile of the lifetimes.
Section 2.4

22. The error in the length of a part (absolute value of the difference between the actual length and the target length), in mm, is a random variable with probability density function

\[ f(x) = \begin{cases} 
  e^{-x} & 0 < x < 1 \\
  0 & \text{otherwise} 
\end{cases} \]

a. What is the probability that the error is less than 0.2 mm?
b. Find the mean error.
c. Find the variance of the error.
d. Find the cumulative distribution function of the error.
e. Find the median error.
f. The specification for the error is 0 to 0.3 mm. What is the probability that the specification is met?

Exercises for Section 2.5

1. If a resistor with resistance \( R \) ohms carries a current of \( I \) amperes, the potential difference across the resistor, in volts, is given by \( V = IR \). The resistance of a randomly selected resistor that is labeled 100 \( \Omega \) has mean 100 \( \Omega \) and standard deviation 10 \( \Omega \). A current of 3 A is set up in a randomly selected resistor.
   a. Find \( \mu_V \).
   b. Find \( \sigma_V \).

2. Two resistors, with resistances \( R_1 \) and \( R_2 \), are connected in series. The combined resistance \( R \) is given by \( R = R_1 + R_2 \). Assume that \( R_1 \) has mean 50 \( \Omega \) and standard deviation 5 \( \Omega \), and that \( R_2 \) has mean 100 \( \Omega \) and standard deviation 10 \( \Omega \).
   a. Find \( \mu_R \).
   b. Assuming \( R_1 \) and \( R_2 \) to be independent, find \( \sigma_R \).

3. A machine that fills cardboard boxes with cereal has a fill weight whose mean is 12.02 oz, with a standard deviation of 0.03 oz. A case consists of 12 boxes randomly sampled from the output of the machine.
   a. Find the mean of the total weight of the cereal in the case.
   b. Find the standard deviation of the total weight of the cereal in the case.
   c. Find the mean of the average weight per box of the cereal in the case.
   d. Find the standard deviation of the average weight per box of the cereal in the case.
   e. How many boxes must be included in a case for the standard deviation of the average weight per box to be 0.005 oz?

Section 4.1

Exercises for Section 4.1

1. At a certain fast-food restaurant, 25% of drink orders are for a small drink, 35% for a medium, and 40% for a large. Let \( X = 1 \) if a randomly chosen order is for a small, and let \( X = 0 \) otherwise. Let \( Y = 1 \) if the order is for a medium, and let \( Y = 0 \) otherwise. Let \( Z = 1 \) if it is for either a small or a medium, and let \( Z = 0 \) otherwise.
   a. Let \( p_X \) denote the success probability for \( X \). Find \( p_X \).
   b. Let \( p_Y \) denote the success probability for \( Y \). Find \( p_Y \).
   c. Let \( p_Z \) denote the success probability for \( Z \). Find \( p_Z \).
   d. Is it possible for both \( X \) and \( Y \) to equal 1?
   e. Does \( p_Z = p_X + p_Y \)?
   f. Does \( Z = X + Y \)? Explain.

2. Let \( X \) and \( Y \) be Bernoulli random variables. Let \( Z = X + Y \).
   a. Show that if \( X \) and \( Y \) cannot both be equal to 1, then \( Z \) is a Bernoulli random variable.
   b. Show that if \( X \) and \( Y \) cannot both be equal to 1, then \( p_Z = p_X + p_Y \).
   c. Show that if \( X \) and \( Y \) can both be equal to 1, then \( Z \) is not a Bernoulli random variable.