Problem #1: Model:

\[
\begin{array}{cccccc}
    x & 3 & 4 & 5 & 6 & 7 & >8 \\
    f(x) & .12 & .19 & .28 & .24 & .10 & .07 \\
\end{array}
\]

\[
P(X \geq 5) = 1 - P(X \leq 4) = 1 - (f(3) + f(4))
\]

\[
= 1 - (.12 + .19) = 1 - .31 = .69
\]

Problem #2: \( n_1 = 200 \) \( n_2 = 500 \)

\[
Y_1 = 120 \quad Y_2 = 240
\]

for town for county

\[
\hat{p}_1 = \frac{120}{200} = .6
\]

\[
\hat{p}_2 = \frac{240}{500} = .48
\]

let

\[
\hat{p} = \frac{Y_1 + Y_2}{n_1 + n_2} = \frac{120 + 240}{200 + 500} = \frac{360}{700} = .5143
\]

\( \hat{p} \) is the proportion of town voters favoring the proposal
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$p_2$ = the proportion of county voters favoring the proposal

**Null Hypothesis**

$H_0: p_1 - p_2 = 0 \text{ vs } H_1: p_1 - p_2 > 0$

**Text Statistic**

$$T = \frac{p_1 - p_2 - (p_1 - p_2)}{\sqrt{\frac{p(1-p)}{n_1 + n_2}}}$$

**Null Distribution (under $H_0$):**

$$T \sim N(0,1)$$

**Rejection Region:**

$$R = [\frac{2}{O.05}, +\infty) = [1.645, +\infty]$$

**Calculations:**

$$T_{obs} = \frac{0.6 - 0.48 - 0}{\sqrt{(0.5143)(1-0.5143)(\frac{1}{200} + \frac{1}{500})}} = \frac{2.8697}{2.8697} = 2.8697$$
Tobias is in R → Ho is rejected in favor of H₁.
that is with level of significance α = 0.05, it appears
the proportion of town voters favoring the proposal is higher
than the county voters.

Problem #3:

\[ \text{white phone} \quad p = 0.20 \]
\[ \text{residents} \quad \text{not-white phone} \quad q = 0.80 \]

\[ n = 1000 \]

Let \( X \) = the number of people choosing a white phone.

Model:

\[ X = \text{Bin}(n=1000, p = 0.20) \]

\[ P(210 \leq X \leq 225) = \sum_{k=210}^{225} \binom{1000}{k} (0.20)^k (0.80)^{1000-k} \]

\[ k = 210 \]
Normal Approximation: Since

\[ np = 1000 \times (0.20) = 200 > 5 \]
\[ nq = 1000 \times (0.80) = 800 > 5 \]

\[ npq = 1000 \times \frac{(20)}{32} \times \frac{(80)}{32} = 160 \]

\[ X \approx \text{Normal}(np, npq) \approx \text{Normal}(\mu=200; \sigma^2=160) \]

Standardized r.v.:

\[ Z = \frac{X - 200}{\sqrt{160}} \]

\[ P(210 \leq X \leq 225) \approx \]
\[ \approx P \left( \frac{210 - 200}{\sqrt{160}} \leq Z \leq \frac{225 - 200}{\sqrt{160}} \right) = \]
\[ = P \left( \frac{7}{4\sqrt{10}} \leq Z \leq \frac{25}{4\sqrt{10}} \right) = \]
\[ = P(0.79 \leq Z \leq 1.98) = \]
\[ = \Phi(1.98) - \Phi(0.79) \]
\[ = 0.9761 - 0.7852 = 0.1909 \]
Problem #4:

**Existing procedure**

- 1st item
  - Def.
  - Not Def.

**New procedure**

- 1st item
  - Def.
  - Not Def.

\( n_1 = 1500 \) items

\( Y_1 = \# \text{ of Defectives out of 1500} \)

\( Y_1 = 75 \)

\( p_1 = \text{the proportion of Defectives} \)

\( n_2 = 2000 \) items

\( Y_2 = \# \text{ of Defectives out of 2000} \)

\( Y_2 = 80 \)

\( p_2 = \text{the proportion of Defectives} \)

Point estimators of \( p_1 \) and \( p_2 \):

\[ \hat{p}_1 = \frac{Y_1}{n_1} = \frac{75}{1500} = 0.05 \]

\[ \hat{p}_2 = \frac{Y_2}{n_2} = \frac{80}{2000} = 0.04 \]
90% CI for $p_1 - p_2$:

$$
(p_1 - p_2) \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}
$$

$\alpha = 1 - .90 = .10$

$\alpha/2 = .05 \quad z_{.05} = 1.645$

$$
CI_{p_1 - p_2}:

(.05 - .04) \pm .1645 \sqrt{\frac{(.05)(1-.05)}{1500} + \frac{(.04)(1-.04)}{2000}}
$$

i.e.

$-.00173 \leq p_1 - p_2 \leq .00173$
Problem #5:

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>X²</th>
<th>XY</th>
<th>Y²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1.3</td>
<td>0.25</td>
<td>0.65</td>
<td>1.69</td>
</tr>
<tr>
<td>1.5</td>
<td>3.4</td>
<td>2.25</td>
<td>5.1</td>
<td>11.56</td>
</tr>
<tr>
<td>3.2</td>
<td>6.7</td>
<td>10.24</td>
<td>21.44</td>
<td>44.89</td>
</tr>
<tr>
<td>4.2</td>
<td>8.0</td>
<td>17.64</td>
<td>33.6</td>
<td>64.0</td>
</tr>
<tr>
<td>5.1</td>
<td>10.0</td>
<td>26.01</td>
<td>51.0</td>
<td>100.0</td>
</tr>
<tr>
<td>6.5</td>
<td>13.2</td>
<td>42.25</td>
<td>85.8</td>
<td>174.24</td>
</tr>
</tbody>
</table>

Totals: 21.0  42.6  98.64  197.59  396.38

\[
\bar{x} = \frac{1}{6} \sum x_i = \frac{1}{6} \cdot (21) = 3.5 \\
\bar{y} = \frac{1}{6} \sum y_i = \frac{1}{6} \cdot (42.6) = 7.1
\]

\[
S_{xx} = \sum (x^2) - \left( \frac{\sum x}{n} \right)^2 = 98.64 - \left( \frac{21.0}{6} \right)^2 = 25.14
\]

\[
S_{xy} = \sum xy - \left( \frac{\sum x \cdot \sum y}{n} \right) = \\
= 197.59 - \frac{21 \cdot 42.6}{6} = 48.49
\]

\[
S_{yy} = \sum (y^2) - \left( \frac{\sum y}{n} \right)^2 = \\
= 396.38 - \frac{(42.6)^2}{6} = 93.92
\]
(a) \[ \hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{48.49}{25.14} = 1.9288 \]

\[ \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = \]

\[ = 7.1 - 1.9288 \times 3.5 = 3.492 \]

(b) \[ \hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x \] - estimated regression line

\[ \hat{y} = 3.492 + 1.9288x \]
Problem #6: \( \frac{O_{ij}}{E_{ij}} \)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>People 1: James</td>
<td>15/11</td>
<td>27/24</td>
<td>16/20</td>
<td>4/6</td>
<td>62</td>
</tr>
<tr>
<td>People 2: Walter</td>
<td>7/11</td>
<td>21/24</td>
<td>24/20</td>
<td>8/6</td>
<td>60</td>
</tr>
</tbody>
</table>

\[ n_1 = 62 \quad n_2 = 60 \]

\[ m_1 = 2.2 \quad m_2 = 4.3 \quad m_3 = 2.4 \quad m_4 = 1.2 \quad N = 122 \]

\( \alpha = 0.01 \)

Null Hypothesis:

\[ H_0 : P_1 = P_2 = P_3 = P_4 \quad \text{Proportions for People 1} \]

\[ H_1 : P_i \neq P_j \quad \text{for at least one pair of } P_i \neq P_j \]

Test Statistic:

\[ T = \sum_{all \ i \neq j} \left( \frac{O_{ij} - E_{ij}}{E_{ij}} \right)^2 \]

Null Distribution: (if all \( E_{ij} \geq 5 \))

\[ T \sim \chi^2 \quad \text{with d.f.} = (2-1) \times (4-1) = 3 \]
Rejection Region:

\[ R = [ \chi^2_{0.01}, 2.5 = 3 + \infty ) = \]

\[ = [11.345, +\infty ) \]

Calculations:

\[ E_{11} = \frac{n_1 \cdot m_1 \cdot 62 \times 22}{N} = \frac{11}{122} \]

\[ E_{12} = \frac{n_1 \times m_2 \cdot 62 \times 48}{N} = \frac{24}{122} \]

\[ E_{13} = \frac{n_1 \times m_3 \cdot 62 \times 40}{N} = \frac{20}{122} \]

\[ E_{14} = \frac{n_1 \times m_4 \cdot 62 \times 12}{N} = \frac{6}{122} \]
$$E_{21} = \frac{n_2 \times m_1}{N} = \frac{60 \times 22}{122} = 10.8 \approx 11$$

$$E_{22} = \frac{n_2 \times m_2}{N} = \frac{60 \times 48}{122} = 23.6 \approx 24$$

$$E_{23} = \frac{n_2 \times m_3}{N} = \frac{60 \times 40}{122} = 19.6 \approx 20$$

$$E_{24} = \frac{n_2 \times m_4}{N} = \frac{60 \times 12}{122} = 5.9 \approx 6$$

$$T_{obs} = \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}} =$$

$$= \frac{(15-11)^2}{11} + \frac{(27-24)^2}{24} + \frac{(16-20)^2}{20} + \frac{(4-6)^2}{6} +$$

$$+ \frac{(7-11)^2}{11} + \frac{(21-24)^2}{24} + \frac{(24-20)^2}{20} + \frac{(8-6)^2}{6} =$$

$$= 6.592$$

$$T_{obs} = 6.592 \not\in [11.345, +\infty)$$

$$H_0$$ is not rejected.

There is no significant diff. between proportions.
Problem #7:  \( n = 5 \), \( \alpha = 0.05 \), level of signif.

Null Hypothesis:
\( H_0: \mu = 0.5 \text{ (karat)} \)

\( \mu \) = mean weight of the diamonds produced

\( \mu_0 = 0.5 \text{ (karat)} \) - null parameter

\( H_1: \mu > 0.5 \text{ (karat)} \), \( (\mu_0 = 0.5) \)

Test Statistic:
\[
T = \frac{X - 0.5}{s/\sqrt{5}}
\]

Null Distribution:

In a \( t \)-distribution with d.f. = 5 - 1 = 4

Rejection Region:
\[
R = [t_{0.05, 4}, +\infty) = [2.132, +\infty)
\]
Calculations:

\[
\bar{X} = \frac{1}{5} \sum_{i=1}^{5} X_i = \frac{1}{5} \left[ 0.46 + 0.61 + 0.52 + 0.48 + 0.54 \right] = 0.522
\]

\[
S^2 = \frac{1}{n-1} \left[ \sum_{i=1}^{5} X_i^2 - \frac{(\sum_{i=1}^{5} X_i)^2}{n} \right]
\]

\[
S^2 = \frac{1}{4} \left[ 1.3761 - \frac{6.7121}{5} \right] = 0.00342
\]

\[
S = \sqrt{0.00342} = 0.05842
\]

\[
T_{\bar{X}} = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{0.522 - 0.5}{0.05842/\sqrt{5}} = 1.8412
\]

\[
T_{\bar{X}} = 1.8412 \not\in [2.132, +\infty)
\]

\(H_0\) is not rejected.

there is insufficient evidence to indicate the average weight of diamonds produced by the process is in excess of 0.5 karat.
Let the lifetime $X$ (in years):

Problem 8: $X \sim \text{Exp} (\beta) \quad \therefore \quad \mathbb{E}(X) = \beta$

Let $X \sim f(x) = \frac{1}{\beta} e^{-\frac{x}{\beta}}, \quad x \geq 0$

with $\beta = 2$

$n = 100$ switches are installed

Prob. that at most 40 switches fail during the first year.

Let $P(X \leq x) = F(x) = \text{cdf of } X$

\[
F(x) = \int_{0}^{x} \frac{1}{\beta} e^{-\frac{t}{\beta}} dt = \int_{0}^{x} \frac{1}{2} e^{-\frac{t}{2}} dt = \]

\[
= 1 - e^{-\frac{x}{2}}
\]

$P(X \leq x)$ is the probability that a switch is failed till $x$

$X$

0

x years

-14-
The probability that a switch is failed during 1 year is
\( P(X \leq 1 \text{ year}) = F(1) = 1 - e^{-t} = 0.39347 \)

After 1 year, the probability of the \( i \)-th switch failing is \( p = 0.39347 \)

Not failed, \( q = 0.60653 \)

Let \( Y = \# \text{ of failed switches out of 100} \) during 1 year

\( Y = \text{Bin}(n=100, \ p=0.39347) \)

\[ P(Y \leq 40) = P \left( \frac{Y - np}{\sqrt{npq}} \leq \frac{40 - 39.347}{4.875} \right) \]

\[ = P \left( Z \leq 0.1336 \right) = \Phi(0.13) \]

\[ = 0.5517 \]