# Review for Test I

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## 1 The Additive Model

### Assumptions

\[ y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \]

with

\[ \sum_i \alpha_i = 0 \quad \text{and} \quad \varepsilon_{ij} \text{IND}(0, \sigma^2) \]

### Three Types of Variance

<table>
<thead>
<tr>
<th>Type of Variance</th>
<th>Formula</th>
<th>Type of SS</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Variance</td>
<td>[ \sum_i \sum_j (y_{ij} - \bar{y})^2 ]</td>
<td>Total SS</td>
<td>[ \sum_i \sum_j (y_{ij} - \bar{y})^2 ]</td>
</tr>
<tr>
<td>Within-group variance</td>
<td>[ \sum_i \sum_j \frac{(y_{ij} - \bar{y}_i)^2}{n_i} ]</td>
<td>Within SS</td>
<td>[ \sum_i \sum_j (y_{ij} - \bar{y}_i)^2 ]</td>
</tr>
<tr>
<td>Among-group variance</td>
<td>[ \frac{n \sum_{i=1} (\bar{y}_i - \bar{y})^2}{a-1} ]</td>
<td>Among SS</td>
<td>[ n \sum_{i=1} (\bar{y}_i - \bar{y})^2 ]</td>
</tr>
</tbody>
</table>

Among-group variance is also called Among-MS (denote MS\(_a\)), and within-group variance is also called Within-MS (denote MS\(_w\)). And pooled variance if determined by

\[ s_p^2 = \frac{\sum_i (n_i - 1)s_i^2}{\sum_i (n_i - 1)} \]

### Test Statistic

\[ F = \frac{\text{among-MS}}{\text{within-MS}} \]

If \( F > q_f(1 - \alpha, df_1 = a - 1, df_2 = a(n - 1)) \), then reject the null hypothesis at a significant level of \( \alpha \).

## 2 Techniques for One-Way Analysis of Variance

Uncorrected total SS \( T = \sum_i \sum_j y_{ij}^2 \), Uncorrected group SS \( A = \sum_i \left( \sum_j y_{ij} \right)^2 / n \), Correction factor \( CF = \left( \sum_i \sum_j y_{ij} \right)^2 / an \). Turns out that Among SS is \( A - CF \) and Within SS is \( T - A \).

With unequal groups, \( T = \sum_i \sum_j y_{ij}^2 \), \( A = \sum_i \left( \sum_j y_{ij} \right)^2 / n_i \), and \( CF = \left( \sum_i \sum_j y_{ij} \right)^2 / N \), \( df_1 = a - 1 \), \( df_2 = N - a \).

## 3 Multiple Comparison Procedures (decide which pairs are different)

Let \( \bar{y}_{(1)}, \ldots, \bar{y}_{(a)} \) be the ordered means of all treatment groups. The difference between \( \bar{y}_{(i)} \) and \( \bar{y}_{(j)} \) are significant w.r.t. \( \alpha \) if \( |\bar{y}_{(i)} - \bar{y}_{(j)}| \geq \)
Fisher’s Least Significant Difference  \[ qt\left(\frac{1-\alpha}{2}, df = a(n-1)\right) \sqrt{\frac{2\text{MS}_e}{n}} \]

Duncan’s New Multiple Range Test  \[ d_{\alpha,|i-j|+1,a(n-1)} \sqrt{\frac{\text{MS}_e}{n}} \]

The Student-Newman-Keuls’ Procedure  \[ q_{\alpha,|i-j|+1,a(n-1)} \sqrt{\frac{\text{MS}_e}{n}} \]

Tukey’s Honestly Significant Difference  \[ d_{\alpha,\alpha,a(n-1)} \sqrt{\frac{\text{MS}_e}{n}} \]

Scheffé’s Method  \[ \sqrt{\frac{(a-1)q_f(1-\alpha, a-1,a(n-1))}{2\text{MS}_e n}} \]

For unequal group sizes, Fisher’s and Scheffé’s procedures apply with the change of standard error \[ \sqrt{\frac{\text{MS}_{ni} + \text{MS}_{nj}}{n_i}} \]. DNMRT, ASNK and HSD apply by letting \[ \tilde{n} = \frac{n}{\sum \frac{1}{n_i}} \]

4 One Degree of Freedom Comparisons

\[ H_0 : \sum_i a_i \mu_i = 0 \]
where \( \sum_i a_i = 0 \) and \( \sum_i a_i b_i = 0 \). The test statistic is

\[ F = \frac{\sum_i T_i^2}{n \sum_i a_i^2} \]

Reject \( H_0 \) if \( F \geq q_f(1-\alpha, df_1 = 1, df_2 = a(n-1)) \).

5 Nonparametric Statistics: Kruskal-Wallis Anova for Ranks

Assumption

\[ E(\bar{r}_i) = \frac{N+1}{2} \forall i \]

Test Statistic

\[ H = \frac{n \left[ \sum_i \left( r_i - \frac{N+1}{2} \right)^2 \right]}{N(N+1)/12} \]

Reject \( H_0 \) if \( H \geq q_{\text{chisq}}(1-\alpha, a-1) \).

For an a posteriori procedure similar to Fisher’s LSD, we use critical value

\[ z_{0.025} \sqrt{\frac{2[N(N+1)]}{12n}} \]

For an orthogonal contrast, the \( H \) value is computed by

\[ H = \frac{\left[ \sum a_i R_i \right]^2 / (n \sum a_i^2)}{N(N+1)/12} \]

Reject if \( H \geq q_{\text{chisq}}(1-\alpha, 1) \).